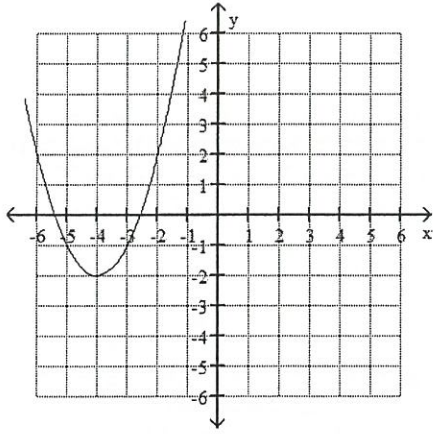Pre–Calculus

1st Semester Review Packet
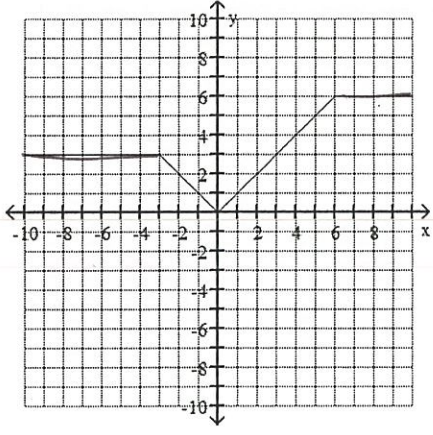
**Use the graph to determine the function's domain and range.**

1)



$D: (-\infty, \infty)$ or $\mathbb{R}$

$R: [-2, \infty)$

2)



$D: (-\infty, \infty)$ or $\mathbb{R}$

$R: [0, 6]$

**Find the domain of the rational function.**

3) $f(x) = \dfrac{x + 3}{x^2 - 4}$

    A) $\{x \mid x \neq -2, x \neq 2, x \neq -3\}$          B) all real numbers

    C) $\{x \mid x \neq -2, x \neq 2\}$                D) $\{x \mid x \neq 0, x \neq 4\}$

**Solve the problem.**

4) The function $P(x) = 0.6x - 73$ models the relationship between the number of pretzels x that a certain vendor sells and the profit the vendor makes. Find P(800), the profit the vendor makes from selling 800 pretzels.

                              $\$407$

5) The owner of a video store has determined that the profits P of the store are approximately given by

    $P(x) = -x^2 + 30x + 73$, where x is the number of videos rented daily. Find the maximum profit to the nearest dollar.

                     $\$298$

For the given functions f and g, find the indicated composition.

6) $f(x) = \dfrac{5}{x+2}$,   $g(x) = \dfrac{2}{5x}$

$(f \circ g)(x)$

$\dfrac{25x}{2+10x}$

## Solve the problem.

7) A discount warehouse offers two types of annual memberships. Plan A has an annual membership fee of $200 and the member pays 50% of the manufacturer's recommended list price. Plan B has an annual membership fee of $70 and the member pays 90% of the manufacturer's recommended list price. How many dollars of merchandise would a member have to purchase in a year to pay the same amount under both plans?

$200 + .5x = 70 + .9x$

$130 = 4x$

$\boxed{x = \$325}$

8) A kennel owner has 600 feet of fencing to enclose a rectangular dog exercise pen. Express the area of the exercise pen, A, as a function of one of its dimensions, x.
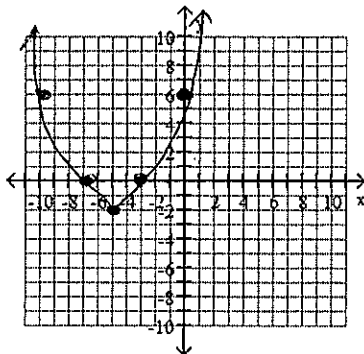
$2x + 2\ell = 600$

$x = \frac{1}{2}(600 - 2\ell) = 300 - \ell$
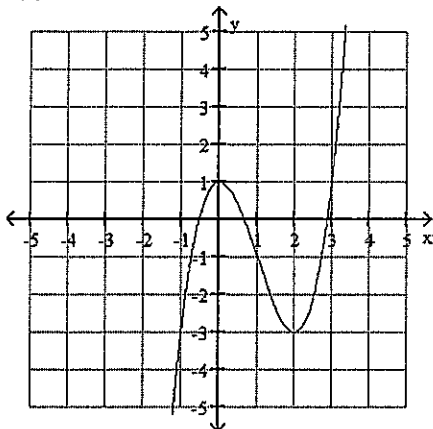
$\boxed{A = x(300 - x)}$

Begin by graphing the standard quadratic function $f(x) = x^2$. Then use transformations of this graph to graph the given function.

9) $g(x) = \dfrac{1}{2}(x+5)^2 - 2$

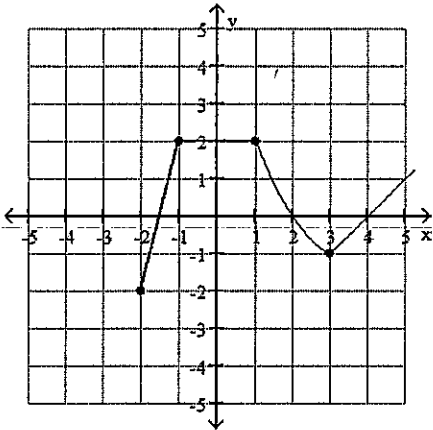Use the graph of the given function to find any relative maxima and relative minima.

10) $f(x) = x^3 - 3x^2 + 1$



Max: at $(0, 1)$
Min: at $(2, -3)$

Identify the intervals where the function is changing as requested.

11) Increasing



$(-2, -1)$ or $(3, \infty)$

Determine the end behavior of the polynomial function.

12) $f(x) = -4x^3 - 2x^2 + 2x + 2$

    A) falls to the left and falls to the right      B) rises to the left and falls to the right

    C) rises to the left and rises to the right      D) falls to the left and rises to the right

Solve the problem.

13) Solve the equation $3x^3 - 20x^2 + 23x + 10 = 0$ given that 2 is a zero of $f(x) = 3x^3 - 20x^2 + 23x + 10$.

$$\{2, 5, -\tfrac{1}{3}\}$$

Solve the polynomial equation.

14) $x^4 - 2x^3 + 6x^2 + 8x - 40 = 0$    $\{-2, 2, 1+3i, 1-3i\}$

15) $f(x) = x^3 + x^2 - 12x$    $\{0, -4, 3\}$

16) $4x^2 = -12x - 6$ $\quad \left\{ \dfrac{-3-\sqrt{3}}{2}, \dfrac{-3+\sqrt{3}}{2} \right\}$

17) $f(x) = 2x^3 - x^2 - 12x + 6$ $\quad \left\{ \dfrac{1}{2}, \sqrt{6}, -\sqrt{6} \right\}$

**Simplify using properties of exponents.**

18) $(4x^{3/4})(4x^{1/2})$ $\qquad 16x^{5/4}$

19) $\dfrac{12x^{2/3}}{4x^{1/4}}$ $\qquad 3x^{5/12}$

**Solve the equations.**

20) $4(7 - 3x) = \dfrac{1}{16}$ $\qquad 3$

21) $e^{3x} = 7$ $\qquad \dfrac{\ln 7}{3}$

22) $9 + 5\ln x = 16$ $\qquad e^{7/5}$

23) $\log_2 3 + \log_2 x = 1$ $\qquad \dfrac{2}{3}$

24) $2\log x = \log 400$ $\qquad 20$

25) $\ln(x - 6) + \ln(x + 1) = \ln(x - 15)$ $\qquad \varnothing$ No solutions

**Solve.**

26) An endangered species of fish has a population that is decreasing exponentially ($A = A_0 e^{kt}$). The population 5 years ago was 1700. Today, only 900 of the fish are alive. Once the population drops below 100, the situation will be irreversible. When will this happen, according to the model? (Round to the nearest whole year.)

$\qquad$ 17 years from today

27) The population of a particular country was 30 million in 1981; in 1992, it was 39 million. The exponential growth function $A = 30e^{kt}$ describes the population of this country t years after 1981. Use the fact that 11 years after 1981 the population increased by 9 million to find k to three decimal places.

$\qquad$ 0.024

28) The value of a particular investment follows a pattern of exponential growth. In the year 2000, you invested money in a money market account. The value of your investment t years after 2000 is given by the exponential growth model $A = 1800e^{0.057t}$. By what percentage is the account increasing each year?
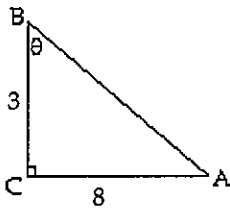
$\qquad$ 5.7%

**Find the exact value of the trigonometric function. Do not use a calculator.**

29) $\sec\dfrac{\pi}{4}$ $\quad = \dfrac{1}{\sqrt{2}/2} = \dfrac{2}{\sqrt{2}} = \dfrac{2\sqrt{2}}{2} = \sqrt{2}$

30) $\tan\left(-\dfrac{5\pi}{4}\right) = -1$

4

Find the indicated trigonometric function of the given angle. Give an exact answer with a rational denominator.
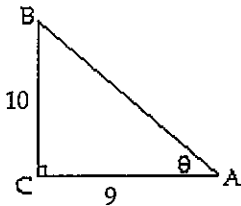
31) Find sec θ.



$$\frac{\sqrt{73}}{3}$$

Then find the indicated trigonometric function of the given angle. Give an exact answer with a rational denominator.

32) Find cos θ.



$$\frac{9\sqrt{181}}{181}$$

Find the exact value of the indicated trigonometric function of θ.

33) $\csc\theta = -\frac{9}{2}$,  θ in quadrant III    Find cot θ.    $\dfrac{\sqrt{77}}{2}$

34) $\cot\theta = -\frac{5}{4}$,  $\cos\theta < 0$    Find csc θ.    $\dfrac{\sqrt{41}}{4}$

**Solve the problem.**

35) A building 270 feet tall casts a 100 foot long shadow. If a person stands at the end of the shadow and looks up to the top of the building, what is the angle of the person's eyes to the top of the building (to the nearest hundredth of a degree)? (Assume the person's eyes are 6 feet above ground level.)

$$69.25°$$

36) A radio transmission tower is 150 feet tall. How long should a guy wire be if it is to be attached 7 feet from the top and is to make an angle of 25° with the ground? Give your answer to the nearest tenth of a foot.

$$338.4 \text{ feet}$$

37) From a boat on the lake, the angle of elevation to the top of a cliff is 32°9'. If the base of the cliff is 1203 feet from the boat, how high is the cliff (to the nearest foot)?

$$756 \text{ feet}$$

38) A surveyor is measuring the distance across a small lake. He has set up his transit on one side of the lake 140 feet from a piling that is directly across from a pier on the other side of the lake. From his transit, the angle between the piling and the pier is 55°. What is the distance between the piling and the pier to the nearest foot?

$$200 \text{ feet}$$

Solve the right triangle shown in the figure. Round lengths to one decimal place and express angles to the nearest tenth of a degree.
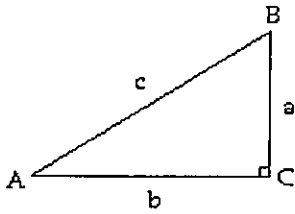


39) A = 49.5°, c = 57.9

$B = 40.5°, \ a = 44, \ b = 37.6$

40) B = 42.4°, c = 4.7 mm

$a = 3.5 \ mm, \ A = 47.6°, \ b = 3.2 \ mm$

41) A = 44°, b = 50.7

$B = 46°, \ a = 49, \ c = 70.5$

42) B = 43°, b = 55.6

$A = 47°, \ a = 59.6, \ c = 81.5$

**Determine the amplitude or period as requested.**

43) Period of $y = -3 \cos \frac{1}{2}x$

$Period = 4\pi$

44) Amplitude of $y = -3 \cos \frac{1}{4}x$

$Amplitude = 3$

**Determine the phase shift of the function.**

45) $y = -4 \cos (8x + \pi)$

$\frac{\pi}{8}$ units to the left

46) $y = -2 \sin \left( \frac{1}{4}x - \frac{\pi}{4} \right)$

$\pi$ units to the right

**Find the exact value of the expression.**

47) $\cos^{-1} \frac{\sqrt{2}}{2}$

$\frac{\pi}{4}$

48) $\sin^{-1} \left( -\frac{1}{2} \right)$

$-\frac{\pi}{6}$

49) $\tan^{-1} \sqrt{3}$

$\frac{\pi}{3}$

50) $\sin^{-1} (0)$

$0$